Implications of a 125 GeV Higgs Boson for the Minimal and Next-To-Minimal Supersymmetric Standard Model

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In March 2013, CERN tentatively announced the discovery of a mass ≈ 125 GeV/c^2 Higgs boson. While the discovery of the Higgs boson is a great triumph for the Standard Model of particle physics, no supersymmetric particles were found, and the mass of 125 GeV has left many physicists at a crossroads as to whether or not Supersymmetry is a valid extension of the Standard Model. The implications of this discovered mass for the Minimal and Next-To-Minimal Supersymmetric Standard Model are investigated. It was found that the MSSM was heavily constrained, although can still effectively describe the 125 GeV mass, whereas the NMSSM is much less restricted and appears to be the most viable candidate as an extension to the Standard Model.

Introduction
The Standard Model (SM) of particle physics is a theory which classifies all currently known subatomic particles and attempts to combine the electromagnetic, weak and strong nuclear forces. It has gained continuous credence due to the discoveries of the top quarks, tau neutrino and the most recent Higgs boson. The Higgs boson of the SM is an excitation of the Higgs field, a fundamental field explaining why some elementary particles have mass; in particular why photons and gluons are massless and why the W and Z bosons (elementary particles mediating the weak nuclear force) are heavy. One problem
arising from this proposal is known as the hierarchy problem, the discrepancy between the weak and gravitational force; namely why the weak force is 32 orders-of-magnitude larger than gravity and why the Higgs boson is much, much lighter than the Planck mass.

As of yet, the best-motivated solution to this problem is Supersymmetry (SUSY), an extension to the SM which proposes a superpartner for every elementary particle - that is, for every fermion there is a superpartner boson with the same mass and internal quantum numbers excluding spin, which differs by a half-integer; and similarly for bosons (Nilles 1984). Note the superpartner of a particle is named by placing the prefix ‘s’ before the particle name. SUSY results in mass cancellations from both fermionic and bosonic particles in the calculation of quantum corrections to the Higgs mass squared. Thus far, the variation of SUSY which has appeared to be most promising is the Minimal Supersymmetric Standard Model (MSSM), wherein two Higgs doublet fields (the minimum number needed) are required to break electroweak symmetry and for radiative corrections.

However, with the announcement of a mass $125.36 \pm 0.37$ (stat) $\pm 0.18$ (syst) GeV ($/c^2$) Higgs boson (CMS Collaboration 2012, ATLAS Collaboration 2014) found at CERN (via the ATLAS and CMS detectors) through the use of the Large Hadron Collider (LHC), the MSSM has been under heavy scrutiny as of late, and as shall be discussed, this has resulted in severe constraining of the model. The Higgs mass discovery has strongly hinted at new dynamics beyond the MSSM, especially the Next-to-Minimal Supersymmetrical Model (NMSSM) which will be shown to have less constraint placed upon it. Both models will be compared and contrasted throughout the course of this review.

**Implications for the MSSM**

Much work has gone into re-defining the MSSM due to the 125 GeV mass of the Higgs boson discovered. Note that all papers cited have utilised programmes such as Fermilab, FeynHiggs, etc to re-create the MSSM under the constraint of this mass (which can all be easily found online are included in the relevant cited papers). All equation
manipulation is carried out through these various programmes; however it is useful to define the formula for the one-loop Higgs mass:

\[ m_h^2 = m_Z^2 c_{2\beta}^2 + \frac{3m_t^4}{4\pi^2 v^2} \left( \log \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right) \]

where

1. \( m_h \) is the Higgs boson mass.

2. \( c_{2\beta}^2 = \cos^2(2\beta) \), where \( \tan(\beta) \) is the ratio of the vacuum expectation values (the average value of an operator in a vacuum) of the Higgs doublet.

3. \( m_t \) is the top quark mass.

4. \( X_t \) is the stop mixing parameter, equal to \( A_t - \mu \cot(\beta) \), \( A \) is the trilinear coupling and \( \mu \) is the SUSY mass parameter which gives the supersymmetric fermionic partner of the Higgs boson its mass.

5. \( M_S = (m_{\tilde{t}_1}m_{\tilde{t}_2})^{1/2} \) where \( m_{\tilde{t}} \) is the top squark mass.

(please see Draper et al. (2012) for the equation above).

There is much general agreement that, due to the 125 GeV of Higgs boson mass, many of the parameters in this equation of the MSSM are severely constrained, notably \( \tan(\beta) \), \( A \) and the mass of the stop quark (Draper et al. 2012, Ellwanger 2012, Heinemeyer, Stal & Weiglein 2012). As argued by Draper et al. (2012), a lower bound is placed on the value of \( \tan(\beta) \) of 3.5, while a mild lower bound is placed on \( M_S \) when the ratio \( X_t/M_S \) is taken into account. See Figure 1 for an example of the parameter constraints in accordance with the Higgs mass (Draper et al. 2012). The lowest value the \( A \)-term and \( M_S \) can take is the modulus of \( X_t \geq 1000 \) GeV and \( M_S \geq 500 \) GeV. Light-scalar restrictions limits the \( A \)-term to at least 1 TeV. In order for SUSY to hold, large \( A \)-terms and heavy scalars are necessary to comply with the 125 Higgs mass. Also, (quark) mixing was allowed for, and as agreed by Hall & Pinner (2012) and Kadastik et al. (2012), maximal-stop mixing was shown to be necessary. This avoids the less-probable multi-TeV stops (Hall, Pinner & Ruderman 2012). Overall, the MSSM is shown to be heavily constrained. Draper et
al. (2012) also notes that were some assumptions relaxed, such as allowing for the NMSSM, the results could change.

The constrained parameters for the MSSM are also investigated by Heinemeyer, Stal & Weiglein (2012), using the FeynHiggs and HiggsBounds code. In this case the lower limit of \( \tan(\beta) \) was found to be 3.2, lower than the 3.5 value calculated by Draper et al. (2012). A high-sensitivity to \( \tan(\beta) \) was found, resulting in a narrowly allowed region for this parameter under the new Higgs mass. The type of Higgs boson discovered at CERN is seemingly that of the SM, a pure scalar of spin 0, with no 'exotic' properties of a SUSY-like Higgs boson being found (CMS Collaboration 2013 ). The type of Higgs boson discovered, and the likelihood of it being of a certain type, is proposed by Heinemeyer, Stal & Weiglein (2012). Interpreting the discovery as a CP-odd Higgs (CP - charge parity) boson is highly disfavoured. A significant parameter space of the MSSM is found to be compatible with a light CP-even Higgs boson, albeit a heavier CP-even boson could exist allowing for low \( M_A \) (pseudoscalar Higgs boson mass) and moderate values of \( \tan(\beta) \). Although admitting constraints, Heinemeyer, Stal & Weiglein (2012) encourage the MSSM. Carina et al. (2012) agree that some MSSM parameters are severely constrained, and that the results obtained at the LHC concerning photon decay rates corresponds to CP-even Higgs mixing.

Certain constrained MSSM theories are found to be disfavoured by Arbey et al. (2012), such as the minimal anomaly and gauge-mediated SUSY-breaking models (two variations of the MSSM which require specific stop energies, and which predict too-light a Higgs boson mass). Also ruled out in the paper is the non-mixing scenario, predicting a mass of only 123 GeV. Typical mixing scenarios only allow very high \( \tan(\beta) \) and \( M_S \) parameters. In general, many types of the MSSM are taken to be disfavourable, yet gravity-mediated constrained MSSM is still viable with large \( A \) and heavy scalar top quarks. This is in contrast to the seemingly much-available parameter space for a 125 GeV Higgs described by Heinemeyer, Stal & Weiglein (2012).
With regards to neutrino-oscillations, an area unincorporated into the SM alone but which SUSY does attempt to explain, Fukuyama & Okada (2002) studied (solar) neutrino oscillation data and attempted to explain it using a subsection of MSSM, the $SO(10)$ model. A more general case of CP-violation phases was examined, unlike the case where CP is not violated which does not agree with observations. It was found that a heavily constrained parameter region of the MSSM cannot be excluded from agreement with experimental data in predicting the neutrino mass matrix, however all parameters except one were fixed and $\tan(\beta)$ was set to 45, a very high value. Again this $\tan(\beta)$ was shown to be very large in order for the MSSM to fit with current knowledge, as mentioned by previous sources.

The same $SO(10)$ model is investigated by Gogoladze & Shafi (2012), who conclude that perfect $t - b - \tau$ Yukawa unification is possible for a Higgs boson of mass 122 - 124 GeV with an uncertainty of $\pm 3$ GeV. However, $\tan(\beta)$ is - as usual - constrained to a high value of 45 - 47.

Conversely, the BABAR Collaboration (2012) produce findings for which the MSSM does not seem to hold. In considering the decay ratios of $\bar{B} \rightarrow D^{*}\bar{\tau}\bar{v}_\tau$, authors show that the expected results cannot be explained by the presence of a type II charged Higgs doublet, for which the MSSM is a specific case. Thus far, the mass result of the Higgs boson has severely limited the MSSM as a complete extension to the SM.

**Implications for the NMSSM**

The Next-to-Minimal SM has an extended (scale-invariant) superpotential compared to the MSSM, with a generalised $\mu$-term (Maniatis 2010). Investigations into the NMSSM indicate it suffers far less parameter-constraint than its counterpart. Maniatis (2010) has concluded that the Higgs boson mass of 125 GeV complies with the NMSSM model without much limitation, resulting in less fine-tuning than the MSSM at low values of $\tan(\beta)$. See Figure 2 for the visualisation of the larger parameter-space available in the NMSSM compared to the MSSM. Figure taken from (Bastero-Gil *et al.* 2000, ...)
Bastero-Gil et al. (2000) also agree that the NMSSM is less restricted than the MSSM.

Ellwanger (2012) outlines the main differences of this model in comparison to Minimal SUSY, with NMSSM giving rise to 3-neutral CP-even Higgs particles. It is concluded that the best way to validate this model is to retrieve data from the LHC which can be explained by NMSSM rather than MSSM. Such findings would include enhanced signal rates in the $\gamma\gamma$ channels, and the discovery of sparticles of mass which “turn out to be incompatible with the necessarily large radiative corrections to the Higgs mass in the MSSM” and the discovery of the lighter CP-even Higgs boson (Ellwanger 2012, p. 7). The fine-tuning necessary for the NMSSM in order to fit with the 125 GeV mass of the Higgs boson was found to be less straining than the MSSM, with 5-10% fine-tuning needed if the mediation scale is low and stop mixing non- maximal (Hall, Pinner & Ruderman 2012). The lack of need for maximal-stop mixing is in stark contrast to all data gathered from simulations of the Minimal SUSY model.

It is noted by Cao et al. (2012, p.1) that the large $A_t$ and top squark mass values of the MSSM are “much ameliorated” in the NMSSM. This is in much agreement with King, Mühleitner, & Nevzorov (2012, p. 28), who investigate the parameters of the model and find that a 125 GeV Higgs is allowed for all mixing below 1 TeV and all stop quark masses, and conclude that “the NMSSM appears to be the best-compromise between naturalness and minimality that

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**Figure 1:** Contour plot of $m_h$ in $\tan(\beta)$ vs. $X_t/M_s$ plane. The stops were set at $m_Q = m_U = 2$ TeV, and the result is only weakly dependent on the stop mass up to $\sim 5$ TeV. The solid curve is $m_h = 125$ GeV with $m_t = 173.2$ GeV. The band around the curve corresponds to $m_h = 123-127$ GeV. Finally the dashed lines correspond to varying $m_t$ from 172-174. The absent dashed contour at left does not exist (light tops and $X_t < 0$ cannot accommodate a 125 GeV Higgs with 2 TeV stop masses).
conclude that the NMSSM is a less-restricted model describing the finding of a Higgs mass of 125 GeV. While the MSSM has been shown to accurately describe the results, albeit it severely constrained, the NMSSM is far less-limited in terms of available parameter space. For example, \( \tan(\beta) \) need not be so high in order to incorporate a SM-like Higgs boson. The data indicates towards new dynamics beyond the MSSM, and at the moment the NMSSM appears to be the best candidate as an extension to the Standard Model.

Before the announcement of the Higgs discovery, the MSSM was considered the best model to explain SUSY. More research and computational effort should be directed towards the NMSSM as of now, and physicists and those with a scientific interest alike wait for March 2015 for the second running of the LHC, for which collisions of up to 13 TeV are planned. Hopefully new results will be obtained which will make clear which version of SUSY is correct, if any, because we are currently at a crossroads between the concept of SUSY and a multiverse in our explanation of the physical world.
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